## Section 1.3 (page 67)

1. 


3.

(a) $0 \quad$ (b) -5
(a) $0 \quad$ (b) About 0.52 or $\pi / 6$
$\begin{array}{llllll}\text { 5. } 8 & \text { 7. }-1 & \text { 9. } 0 & \text { 11. } 7 & \text { 13. } 2 & \text { 15. } 1\end{array}$
$\begin{array}{llllll}\text { 17. } 1 / 2 & \text { 19. } 1 / 5 & \text { 21. } 7 & \text { 23. (a) } 4 & \text { (b) } 64 & \text { (c) } 64\end{array}$
25. (a) 3
(b) 2
(c) 27.1

29
31. 1
33. $1 / 2 \quad$ 35. $-1 \quad$ 37. (a) 10
(b) 5
(c) 6
(d) $3 / 2$
39. (a) 64
(b) 2
(c) 12
(d) 8
41. (a) -1
(b) -2
$g(x)=\frac{x^{2}-x}{x}$ and $f(x)=x-1$ agree except at $x=0$.
43. (a) 2 (b) 0
$g(x)=\frac{x^{3}-x}{x-1}$ and $f(x)=x^{2}+x$ agree except at $x=1$.
45. -2
$f(x)=\frac{x^{2}-1}{x+1}$ and $g(x)=x-1$ agree except at $x=-1$.
47. 12
$f(x)=\frac{x^{3}-8}{x-2}$ and $g(x)=x^{2}+2 x+4$ agree except at $x=2$.
49. -1
51. $1 / 8$
53. $5 / 6$
55. $1 / 6$
57. $\sqrt{5} / 10$
59. $-1 / 9$
61. 2
63. $2 x-2$
65. $1 / 5$
67. 0
69. 0
71. 0
73. 1
75. $3 / 2$
77.


Answers will vary. Example:

| $\boldsymbol{x}$ | -0.1 | -0.01 | -0.001 | 0.001 | 0.01 | 0.1 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ | 0.358 | 0.354 | 0.354 | 0.354 | 0.353 | 0.349 |

$\lim _{x \rightarrow 0} \frac{\sqrt{x+2}-\sqrt{2}}{x} \approx 0.354\left(\right.$ Actual limit is $\frac{1}{2 \sqrt{2}}=\frac{\sqrt{2}}{4}$.)
79.


Answers will vary. Example:

| $\boldsymbol{x}$ | -0.1 | -0.01 | -0.001 |
| :--- | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ | -0.263 | -0.251 | -0.250 |


| $\boldsymbol{x}$ | 0.001 | 0.01 | 0.1 |
| :--- | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{x})$ | -0.250 | -0.249 | -0.238 |

$\lim _{x \rightarrow 0} \frac{[1 /(2+x)]-(1 / 2)}{x} \approx-0.250\left(\right.$ Actual limit is $\left.-\frac{1}{4}.\right)$
81.


The graph has a hole at $t=0$.

Answers will vary. Example:

| $\boldsymbol{t}$ | -0.1 | -0.01 | 0 | 0.01 | 0.1 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{f}(\boldsymbol{t})$ | 2.96 | 2.9996 | $?$ | 2.9996 | 2.96 |

$\lim _{t \rightarrow 0} \frac{\sin 3 t}{t}=3$
83.


The graph has a hole at $x=0$.

Answers will vary. Example:

| $\boldsymbol{x}$ | -0.1 | -0.01 | -0.001 | 0 | 0.001 | 0.01 | 0.1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\boldsymbol{f}(\boldsymbol{x})$ | -0.1 | -0.01 | -0.001 | $?$ | 0.001 | 0.01 | 0.1 |

$$
\lim _{x \rightarrow 0} \frac{\sin x^{2}}{x}=0
$$

85. 
86. $-1 /(x+3)^{2}$
87. 



0
89. 4
93.


0
95.


0
The graph has a hole at $x=0$.
97. $f$ and $g$ agree at all but one point if $c$ is a real number such that $f(x)=g(x)$ for all $x \neq c$.
99. An indeterminate form is obtained when evaluating a limit using direct substitution produces a meaningless fractional form, such as $\frac{0}{0}$.
101.


The magnitudes of $f(x)$ and $g(x)$ are approximately equal when $x$ is close to 0 . Therefore, their ratio is approximately 1 .
103. $-64 \mathrm{ft} / \mathrm{sec}($ speed $=64 \mathrm{ft} / \mathrm{sec})$
105. $-29.4 \mathrm{~m} / \mathrm{sec}$
107. Let $f(x)=1 / x$ and $g(x)=-1 / x$.
$\lim _{x \rightarrow 0} f(x)$ and $\lim _{x \rightarrow 0} g(x)$ do not exist. However,
$\lim _{x \rightarrow 0}[f(x)+g(x)]=\lim _{x \rightarrow 0}\left[\frac{1}{x}+\left(-\frac{1}{x}\right)\right]=\lim _{x \rightarrow 0} 0=0$
and therefore does exist.
109-113. Proofs
115. Let $f(x)=\left\{\begin{array}{rr}4, & \text { if } x \geq 0 \\ -4, & \text { if } x<0\end{array}\right.$
$\lim _{x \rightarrow 0}|f(x)|=\lim _{x \rightarrow 0} 4=4$
$\lim _{x \rightarrow 0} f(x)$ does not exist because for $x<0, f(x)=-4$ and for $x \geq 0, f(x)=4$.
117. False. The limit does not exist because the function approaches 1 from the right side of 0 and approaches -1 from the left side of 0. (See graph below.)


## 119. True.

121. False. The limit does not exist because $f(x)$ approaches 3 from the left side of 2 and approaches 0 from the right side of 2 . (See graph below.)

122. Proof
123. (a) All $x \neq 0, \frac{\pi}{2}+n \pi$
(b)


The domain is not obvious. The hole at $x=0$ is not apparent from the graph.
(c) $\frac{1}{2}$
(d) $\frac{1}{2}$
127. The graphing utility was not set in radian mode.

